

Recent Advances in Semileptonic B Decays

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Abstract

Aspects of the OPE-based QCD theory of B decays are discussed. We have at least one nontrivial precision check of the OPE at the nonperturbative level in inclusive decays. The data suggest proximity to the ‘BPS’ limit for B mesons. Its consequences are addressed and their accuracy qualified. It is suggested that theory-wise $B \rightarrow D \ell \nu$ near zero recoil offers an accurate alternative way to measure $|V_{cb}|$. It is shown that the OPE is in a good standing when confronted by experiment. The alleged controversy between theory and data on certain decay channels is found to be an artefact of oversimplifying the OPE in presence of high experimental cuts severely degrading the effective hardness. The effects exponential in the latter are missed in the traditionally used expressions, yet they do not signify a breakdown of the $1/m_b$ expansion proper. They typically increase the apparent b quark mass in $B \rightarrow X_s + \gamma$ by 70 MeV or more, together with an even more dramatic downward shift in the kinetic expectation value. The utility of the second moment of E_γ is emphasized once the aforementioned effects have been included. Incorporating them brings different measurements into a good agreement, provided the OPE-based theory employs the robust approach.

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1 Introduction

The QCD-based theory of inclusive heavy flavor decays by now has made a long journey from its first fundamental results beginning with the QCD theorem [1] which established absence of the leading Λ_{QCD}/m_b power corrections to total decay rates. Based on the Wilsonian OPE, the heavy quark expansion is now a well developed field of QCD with many nontrivial phenomenological applications [2, 3]. The new generation of data provides accurate measurements of many inclusive characteristics in B decays. In practical terms,

- An accurate and reliable determination of some heavy quark parameters is available directly from experiment;
- Extracting $|V_{cb}|$ from $\Gamma_{\text{sl}}(B)$ has good accuracy and solid grounds;
- We have at least one nontrivial precision check of the OPE at the nonperturbative level.

The latter comes comparing the average lepton energy with the invariant hadronic mass, as explained in Sect. 3.

The present theory allows to aim for a percent accuracy in $|V_{cb}|$. Such a precision becomes possible owing to a number of theoretical refinements. The low-scale running masses $m_b(\mu)$, $m_c(\mu)$, the expectation values $\mu_\pi^2(\mu)$, $\mu_G^2(\mu)$, ... are completely defined and can be determined from experiment with an in principle unlimited accuracy. Violation of local duality potentially limiting theoretical predictability, has been scrutinized and found to be negligibly small in total semileptonic B widths [4]; it can be controlled experimentally with dedicated high-statistics measurements. Present-day perturbative technology makes computing α_s -corrections to the Wilson coefficients of nonperturbative operators feasible. It is also understood how to treat higher-order power corrections in a way which renders them suppressed [5].

The original motivation for the precision control of strong interaction effects in B decays was the quest for $|V_{cb}|$ and $|V_{ub}|$ as means to cross check the Standard Model. Interesting physics lies, however not only in the CKM matrix; knowledge of heavy quark masses and nonperturbative parameters of QCD is of high importance as well.

Some of the heavy quark parameters like μ_G^2 are known beforehand. Proper field-theoretic definition allows its accurate determination from the $B^* - B$ mass splitting: $\mu_G^2(1 \text{ GeV}) = 0.35^{+0.03}_{-0.02} \text{ GeV}^2$ [6]. A priori less certain is μ_π^2 . However, the inequality $\mu_\pi^2 > \mu_G^2$ valid for any definition of kinetic and chromomagnetic operators respecting the QCD commutation relation $[D_j, D_k] = -ig_s G_{jk}$, and the corresponding sum rules essentially limit its range: $\mu_\pi^2(1 \text{ GeV}) = 0.45 \pm 0.1 \text{ GeV}^2$.

The important feature of the heavy quark parameters – in particular the nonperturbative ones – is that they represent well-defined physical quantities which take universal values independent of a process under consideration. As such they can be determined separately in quite different processes with b quarks. The best example is m_b itself – it can be measured through the near-threshold production $e^+e^- \rightarrow b\bar{b}$, or in inclusive B decays. The extracted value must be the same, for it is just an actual b quark mass in actual QCD.

Recent experimental data on inclusive B decays generally show a nontrivial agreement between quite different – and *a priori* unrelated – measurements at the nonperturbative level, on one hand, and consistency with the QCD-based OPE theory. It is fair to note that some discrepancies have been reported as constituting obvious problems for the OPE. In my opinion, however, these claims are unjustified and rather rooted in a sloppy application of the heavy

quark expansion, in particular of the OPE. The proper treatment, in fact seems to yield a good agreement between different measurements, well within correctly assessed theoretical accuracy. I shall illustrate this later in Sect. 3. At this point it is interesting to note that all data seem to point at a relatively low kinetic expectation value $\mu_\pi^2 \lesssim 0.4 \text{ GeV}^2$; the individual uncertainties are sizable, though.

2 BPS limit

An intriguing theoretical environment opens up if $\mu_\pi^2(1 \text{ GeV})$ is eventually confirmed to be close enough to $\mu_G^2(1 \text{ GeV})$ as currently suggested by experiment, say it does not exceed 0.45 GeV^2 . If $\mu_\pi^2 - \mu_G^2 \ll \mu_\pi^2$ it is advantageous to analyze strong dynamics expanding around the point $\mu_\pi^2 = \mu_G^2$ [6]. This is not just one point of a continuum in the parameter space, but a quite special ‘BPS’ limit where the heavy flavor ground state satisfies functional relations $\vec{\sigma}\vec{\pi}|B\rangle=0$. This limit is remarkable in many respects; for example, it saturates the bound [7] $\varrho^2 \geq \frac{3}{4}$ for the slope of the IW function. It should be recalled that already quite some time ago there were dedicated QCD sum rules estimates of both ϱ^2 [8] and μ_π^2 [9] yielding literal values nearly at the respective lower bounds, supporting this limit.

The SV heavy quark sum rules place a number of important constraints on the nonperturbative parameters. For instance, they yield a bound on the IW slope

$$\mu_\pi^2 - \mu_G^2 = 3\tilde{\varepsilon}^2(\varrho^2 - \frac{3}{4}), \quad 0.4 \text{ GeV} \lesssim \tilde{\varepsilon} \lesssim 1 \text{ GeV} \quad (1)$$

so that ϱ^2 can barely reach 1 being rather closer to 0.85. It is interesting that this prediction [6] turned out in a good agreement with the recent lattice calculation [10] $\varrho^2 = 0.83_{-0.11}^{+0.15+0.24}$. It leaves only a small window for the slope of the actual $B \rightarrow D^*$ formfactor, excluding values of $\hat{\varrho}^2$ in excess of 1.15–1.2. This would be a very constraining result for a number of experimental studies, in particular for extrapolating the $B \rightarrow D^*$ rate to zero recoil. Since there is a strong correlation between the extrapolated rate and the slope, this may change the extracted value of $|V_{cb}|$. Therefore, it is advantageous to analyze the $B \rightarrow D^* \ell \nu$ data including the above constraint as an option, and I suggest our experimental colleagues explore this in future analyses.

The experimentally measured slope $\hat{\varrho}^2$ differs from ϱ^2 by heavy quark symmetry-violating corrections. The estimate by Neubert that $\hat{\varrho}^2$ is smaller than ϱ^2 , $\hat{\varrho}^2 \simeq \varrho^2 - 0.09$ seems to be ruled out by experiment. It is not clear if a better estimate can be made in a trustworthy way.

The whole set of the heavy quark sum rules is even more interesting. Their constraining power depends strongly on the actual value of μ_π^2 . When it is at the lower end of the allowed interval, the BPS expansion appears the most effective way to analyze all the relations.

2.1 Miracles of the BPS limit

A number of useful relation for nonperturbative parameters hold in the limit $\mu_\pi^2 = \mu_G^2$: they include $\varrho^2 = \frac{3}{4}$, $\overline{\Lambda} = 2\overline{\Sigma}$, $\rho_{LS}^3 = -\rho_D^3$, relations for nonlocal correlators $\rho_{\pi G}^3 = -2\rho_{\pi\pi}^3$, $\rho_A^3 + \rho_{\pi G}^3 = -(\rho_{\pi\pi}^3 + \rho_S^3)$, etc.

This limit also extends a number of the heavy flavor symmetry relations for the ground-state mesons to all orders in $1/m$:

- There are no formal power corrections to the relation $M_P = m_Q + \bar{\Lambda}$ and, therefore to $m_b - m_c = M_B - M_D$. The routinely used spin-averaged mass difference, however is not stable.
- For the $B \rightarrow D$ amplitude the heavy quark limit relation between the two formfactors

$$f_-(q^2) = -\frac{M_B - M_D}{M_B + M_D} f_+(q^2) \quad (2)$$

does not receive power corrections.

- For the zero-recoil $B \rightarrow D$ amplitude all δ_{1/m^k} terms vanish.
- For the zero-recoil formfactor f_+ controlling decays with massless leptons

$$f_+((M_B - M_D)^2) = \frac{M_B + M_D}{2\sqrt{M_B M_D}} \quad (3)$$

holds to all orders in $1/m_Q$.

- At arbitrary velocity power corrections in $B \rightarrow D$ vanish,

$$f_+(q^2) = \frac{M_B + M_D}{2\sqrt{M_B M_D}} \xi\left(\frac{M_B^2 + M_D^2 - q^2}{2M_B M_D}\right) \quad (4)$$

so that the $B \rightarrow D$ decay rate directly yields Isgur-Wise function $\xi(w)$.

It is interesting that experimentally the slope of the $B \rightarrow D$ amplitude is indeed smaller centering around $\hat{\rho}_{(D)}^2 \approx 1.15$ [11], indicating qualitative agreement with the BPS regime.

What about the $B \rightarrow D^*$ amplitude, are the corrections suppressed as well? Unfortunately, the answer is negative. The structure of power corrections indeed simplifies in the BPS limit, however δ_{1/m^2} , δ_{1/m^3} are still very significant [6], and the literal estimate for $F_{D^*}(0)$ falls even below 0.9. Likewise, we expect too significant corrections to the shape of the $B \rightarrow D^*$ formfactors. Heavy quark **spin** symmetry controlling these transitions seems to be violently affected by strong interactions for charm.

A physical clarification must be added at this point. Absence of all power corrections in $1/m_Q$ for certain relations may be naively interpreted as implying that they would hold for arbitrary, even small quark masses, say in $B \rightarrow K$ transitions. This is not correct, though, for the statement refers only to a particular fixed order in $1/m_Q$ expansion in the strict BPS limit. In fact the relations become more and more accurate approaching this limit only above a certain mass scale of order $\bar{\Lambda}$, while below it their violation is of order unity regardless of proximity of the heavy quark ground state to BPS.

2.2 Quantifying deviations from BPS

Since the BPS limit cannot be exact in actual QCD, it is important to understand the accuracy of its predictions. The dimensionless parameter describing the deviation from BPS is

$$\beta = \|\pi_0^{-1}(\vec{\sigma}\vec{\pi})|B\rangle\| \equiv \sqrt{3\left(\varrho^2 - \frac{3}{4}\right)} = 3\left(\sum_n |\tau_{1/2}^{(n)}|^2\right)^{\frac{1}{2}}. \quad (5)$$

Numerically β is not too small, similar in size to generic $1/m_c$ expansion parameter, and the relations violated to order β may in practice be more of a qualitative nature only. However, the expansion parameters like $\mu_\pi^2 - \mu_G^2 \propto \beta^2$ can be good enough. Moreover, we can count together powers of $1/m_c$ and β to judge the real quality of a particular heavy quark relation. Therefore understanding at which order in β the BPS relations get corrections is required. In fact, we need classification in powers of β to all orders in $1/m_Q$.

Relations (2) and (4) for the $B \rightarrow D$ amplitudes at arbitrary velocity can get first order corrections in β . Thus they may be not very accurate. The same refers to equality of $\rho_{\pi G}^3$ and $-2\rho_{\pi\pi}^3$. The other relations mentioned for heavy quark parameter are accurate up to order β^2 . The other important BPS relations hold up to order β^2 as well:

- $M_B - M_D = m_b - m_c$ and $M_D = m_c + \bar{\Lambda}$
- Zero recoil matrix element $\langle D | \bar{c} \gamma_0 b | B \rangle$ is unity up to $\mathcal{O}(\beta^2)$
- Experimentally measured $B \rightarrow D$ formfactor f_+ near zero recoil receives only second-order corrections in β to all orders in $1/m_Q$:

$$f_+ \left((M_B - M_D)^2 \right) = \frac{M_B + M_D}{2\sqrt{M_B M_D}} + \mathcal{O}(\beta^2). \quad (6)$$

This is an analogue of the Ademollo-Gatto theorem for the BPS expansion.

The similar statement then applies to f_- as well, and the heavy quark limit prediction for f_-/f_+ must be quite accurate near zero recoil. It can be experimentally checked in the decays $B \rightarrow D \tau \nu_\tau$.

As a practical application of the results based on the BPS expansion, one can calculate the $B \rightarrow D$ decay amplitude near zero recoil to use this channel for the model-independent extraction of $|V_{cb}|$ in future high-luminosity experiments. For power corrections we have

$$\frac{M_B + M_D}{2\sqrt{M_B M_D}} f_+ \left((M_B - M_D)^2 \right) = 1 + \left(\frac{\bar{\Lambda}}{2} - \bar{\Sigma} \right) \left(\frac{1}{m_c} - \frac{1}{m_b} \right) \frac{M_B - M_D}{M_B + M_D} - \mathcal{O} \left(\frac{1}{m_Q^2} \right). \quad (7)$$

We see that this indeed is of the second order in β . Moreover, $\bar{\Lambda} - 2\bar{\Sigma}$ is well constrained through $\mu_\pi^2 - \mu_G^2$ by spin sum rules. Including perturbative corrections (which should be calculated in the proper renormalization scheme respecting BPS regime) we arrive at the estimate

$$\frac{2\sqrt{M_B M_D}}{M_B + M_D} f_+(0) = 1.03 \pm 0.025 \quad (8)$$

It is valid through order $\beta^2 \frac{1}{m_c}$ accounting for all powers of $1/m_Q$ to order β^1 . Assuming the counting rules suggested above this corresponds to the precision up to $1/m_Q^4$, essentially better than the “gold-plated” $B \rightarrow D^*$ formfactor where already $1/m_Q^2$ terms are large and not well known. Therefore, the estimate (8) must be quite accurate. In fact, the major source of the uncertainty seems to be perturbative corrections, which can be refined in the straightforward way compared to decade-old calculations.

3 Inclusive B decays, OPE and heavy quark parameters

Inclusive distributions in semileptonic and radiative decays are the portal to accurately determining the nonperturbative heavy quark parameters controlling short-distance observables in

B decays. In this way one can extract the most precise and model-independent value of V_{cb} from Γ_{sl} , and possibly of V_{ub} if a number of experimental problems can be overcome.

High accuracy can be achieved in a comprehensive approach where many observables are measured in B decays to extract necessary ‘theoretical’ input parameters. It is crucial that here one can do without relying on charm mass expansion at all, i.e. do not assume charm quark to be really heavy in strong interaction mass scale. For reliability of the $1/m_c$ expansion is questionable. Already in the $1/m_Q^2$ terms one has $\frac{1}{m_c^2} > 14 \frac{1}{m_b^2}$; even for the worst mass scale in the width expansion, $\frac{1}{(m_b - m_c)^2}$ is at least 8 times smaller than $\frac{1}{m_c^2}$. There are indications [6] that the nonlocal correlators affecting meson masses can be particularly large – a pattern independently observed in the ‘t Hooft model [12]. This expectation is supported by the pilot lattice study [13] which – if taken at face value – suggests a very large value of a particular combination $\rho_{\pi\pi}^3 + \rho_S^3$ entering in the conventional approach. On the other hand, non-local correlators are not measured in inclusive B decays, so that assumptions about them can hardly be verified.

The approach which is free from relying on charm mass expansion [14] was put forward at the CKM-2002 Workshop at CERN. It allows to utilize the full power of the comprehensive studies, and makes use of a few key facts [15, 16]:

- Total width to order $1/m_b^3$ is affected by a single new Darwin operator; the moments depend also on ρ_{LS}^3 , albeit weakly.
- No nonlocal correlators ever enter *per se*.
- Deviations from the HQ limit in the expectation values are driven by the maximal mass scale, $2m_b$ (and are additionally suppressed by proximity to the BPS limit); they are negligible in practice.
- Exact sum rules and inequalities hold for properly defined Wilsonian parameters.

The strategy and how it works in practice has been described in detail in Refs. [14, 17]. Here I only recall a few salient facts.

The gross features of the lepton energy distribution are mainly shaped by the parton expressions and get only slightly corrected by nonperturbative effects. Consequently, a few lowest lepton moments depend primarily on one and the same combination of the heavy quark masses, $m_b - 0.7m_c$, with insufficient sensitivity to the nonperturbative operators. This does not allow in practice to constrain more than one combination of the quark masses. Yet even these moments turn out informative for V_{cb} , since Γ_{sl} depends on nearly the same combination of m_b and m_c .

Moments of invariant mass square M_X^2 in the final state of semileptonic B decays appear more important in constraining the nonperturbative parameters, since it is *ab initio* dominated by nonperturbative terms. However, the first hadronic moment $\langle M_X^2 \rangle$ turns out to depend on practically the same combination $m_b - 0.7m_c + 0.1\mu_\pi^2 - 0.2\rho_D^3$ as the lepton moment. (Here and below all coefficients are assumed to be in the corresponding powers of GeV.) Not very constraining, this provides, however a highly nontrivial check of the HQ expansion. For example, taking DELPHI’s central value for $\langle M_X^2 \rangle$ we would predict $\langle E_\ell \rangle = 1.375$ GeV, while experimentally they obtain $\langle E_\ell \rangle = (1.383 \pm 0.015)$ GeV [18]. In this respect such a comparison is more critical than among the lepton moments themselves. In particular, these two first moments together verify the heavy quark sum rule for $M_B - m_b$ with the accuracy about 40 MeV!

The higher hadronic moments (with respect to the average) are already more directly sen-

sitive to nonperturbative parameters μ_π^2 and ρ_D^3 . Ideally, they would measure the kinetic and Darwin expectation values separately. At present, however, we have only an approximate evaluation and informative upper bound on $\tilde{\rho}_D^3$. The current sensitivity to μ_π^2 and $\tilde{\rho}_D^3$ is about 0.1 GeV^2 and 0.1 GeV^3 , respectively. A more accurate measurement would also require refined theoretical description which is possible once the modified higher hadronic moments are measured at B -factories [17].

The experimental constraints on the combination driving $\Gamma_{\text{sl}}(B)$ appear stronger for the hadronic moment. Using it instead of $\langle E_\ell \rangle$ we would arrive at

$$\frac{|V_{cb}|}{0.042} = 1 + 0.14 [\langle M_X^2 \rangle - 4.54 \text{ GeV}^2] - 0.03 (m_c - 1.15 \text{ GeV}) + 0.1 (\mu_\pi^2 - 0.4 \text{ GeV}^2) + 0.1 (\tilde{\rho}_D^3 - 0.12 \text{ GeV}^3).$$

One finds that measuring the second and third hadronic moments which directly constrain μ_π^2 and $\tilde{\rho}_D^3$ is an essential step in implementing the comprehensive program of extracting $|V_{cb}|$ (see figures in [19, 18]). To illustrate, neglecting possible theoretical uncertainties in the above relations, we get

$$|V_{cb}| = 0.0418 (1 \pm 0.01_{\text{SL width}} \pm 0.02_{\text{HQ par}}) \quad (9)$$

from only the DELPHI hadronic moments. It is crucial that this extraction carries no hidden assumptions, and at no point we rely on $1/m_c$ expansion. Charm quark could be either heavy, or light as strange or up quark, without deteriorating – and rather improving – the accuracy.

Incorporating other measurements further decreases the experimental uncertainty associated with the heavy quark parameters. However, another limiting factor becomes important – the accuracy of the theoretical expression for the moments used in the analyses. All the theoretical expressions relied upon have a limited accuracy, a fact discarded above for simplicity.

A similar analysis can be applied to the moments with a cut on lepton energy; in particular, CLEO measured a few lepton energy moments for $E_\ell > 1.5 \text{ GeV}$ with unprecedented accuracy. They also fix more or less the same combination of masses and nonperturbative parameters. The value comes out close, but does not literally coincide with that obtained by DELPHI. This sometimes is referred to as a problem for theory. My opinion is different – we do not see here a convincing evidence of the theory failure: the intrinsic theory accuracy is essentially degraded by the relatively high cut employed, with the effective hardness deteriorated. The additional theoretical uncertainties are typically overlooked here.

3.1 Cuts and extracting heavy quark parameters

In order to enjoy in full the potential of a small expansion parameter provided by the heavy quark mass, the observable in question must be sufficiently inclusive. However experimental cuts imposed for practical reasons – to suppress backgrounds etc. – often essentially degrade the effective hardness Q of the process. This brings in another expansion parameter $1/Q$ effectively replacing $1/m_b$ in certain QCD effects. The reliability of the expansion greatly deteriorates for $Q \ll m_b$. This phenomenon is particularly important in $b \rightarrow s + \gamma$ decays where experiments so far have imposed cut $E_\gamma > 2 \text{ GeV}$ or even higher.

The theoretical aspects of such limitations have been discussed during the last couple of years [20, 14, 17]. In particular, the effective ‘hardness’ of the inclusive $b \rightarrow s + \gamma$ decays

with $E_\gamma > 2 \text{ GeV}$ amounts to only about 1.25 GeV , which casts doubts on the precision of the routinely used expressions incorporated into the fits of heavy quark parameters.

Ref. [21] has analyzed the numerical aspects and it was shown that these effects turn out significant, lead to a systematic bias that often exceeds naive error estimates and therefore cannot be ignored. Evaluating them in the most straightforward (although somewhat simplified) way we find, for instance for $b \rightarrow s + \gamma$ decays

$$\begin{aligned}\tilde{m}_b &\simeq m_b + 70 \text{ MeV} \\ \tilde{\mu}_\pi^2 &\simeq \mu_\pi^2 - (0.15 \div 0.2) \text{ GeV}^2\end{aligned}\tag{10}$$

where \tilde{m}_b and $\tilde{\mu}_\pi^2$ are the **apparent** values of the b quark mass and of the kinetic expectation value, respectively, as extracted from the $b \rightarrow s + \gamma$ spectrum in a usual way with $E_\gamma > 2 \text{ GeV}$. Correcting for this bias eliminates alleged problems for the OPE in describing different data and rather leads to a too good agreement between the data on different types of inclusive decays.

Moreover, this resolves the controversy noted previously: while the values of $\bar{\Lambda}$ and μ_π^2 reportedly extracted from the CLEO $b \rightarrow s + \gamma$ spectrum were found to be significantly below the theoretical expectations, the theoretically obtained spectrum itself turned out to yield a good description of the observed spectrum when we evaluated it based on these theoretically preferred values of parameters [20].

The origin of these effects and why they are routinely missed in the standard application of the OPE and in estimates of the theoretical accuracy, have been discussed in detail in Ref. [17]. Conceptually this is related to the limited range of convergence of the OPE for the width, determined in this case by the support of the heavy quark distribution function. Referring to the original publications [17, 21] for details, here I give only the numerical estimates for the cut-induced bias in $b \rightarrow s + \gamma$ decays. Namely, we can compare the **apparent** values of the b quark mass \tilde{m}_b and of the kinetic operator $\tilde{\mu}_\pi^2$ with the true ones m_b , μ_π^2 which would be measured if no cut were imposed, Fig. 1. The deficit in $\bar{\Lambda}$ turns out quite significant.

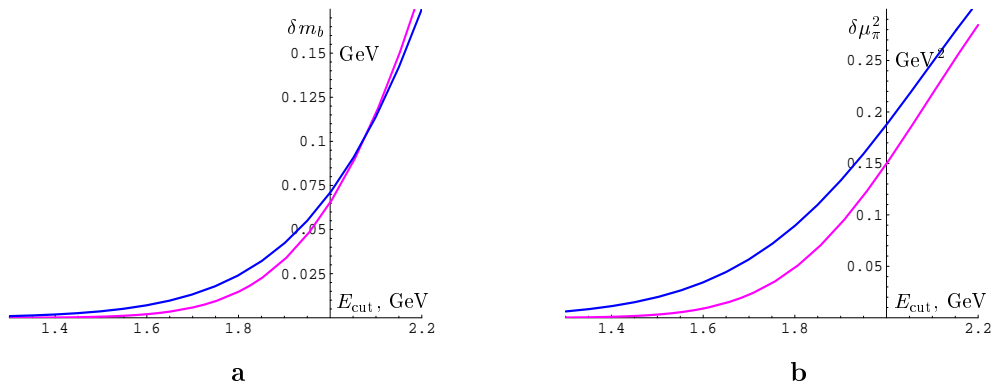


Figure 1: The shifts $\tilde{m}_b - m_b$ in the quark mass (a) and $\mu_\pi^2 - \tilde{\mu}_\pi^2$ in the kinetic operator (b) introduced by imposing a lower cut in the photon energy in $B \rightarrow X_s + \gamma$. Blue and maroon curves correspond to two different ansätze for the heavy quark distribution function.

The naive extraction of the kinetic expectation value through the variance of the truncated distribution undercounts it even more dramatically as Fig. 1b illustrates, since higher moments are more sensitive to the tail of the distribution.

Why are the effects of the cut so large? They are exponential in the inverse hadronic scale μ_{hadr} , but are governed by the **hardness** $\mathcal{Q} \simeq m_b - 2E_{\text{cut}}$ rather than by m_b :

$$\bar{\Lambda} - \tilde{\Lambda}(E_{\text{cut}}) \propto \mu_{\text{hadr}} e^{-\frac{\mathcal{Q}}{\mu_{\text{hadr}}}}, \quad \mu_\pi^2 - \tilde{\mu}_\pi^2 \propto \mu_{\text{hadr}}^2 e^{-\frac{\mathcal{Q}}{\mu_{\text{hadr}}}} \quad (11)$$

(the exponent may have a form of a power of $\mathcal{Q}/\mu_{\text{hadr}}$). Even at $m_b \rightarrow \infty$ these effects survive unless \mathcal{Q} is made large as well!

From this comparison we conclude:

- the value of m_b as routinely extracted from the $b \rightarrow s + \gamma$ spectrum is to be decreased by an amount of order 70 MeV;
- relative corrections to μ_π^2 are even more significant and can naturally constitute a shift of 0.2 GeV^2 . This arises on top of other potential effects.

3.2 Practical implications

One can imagine a dedicated analysis of the consistency of the OPE predictions for inclusive B decays through a simultaneous fit of all available experimental data with the underlying heavy quark parameters. For illustrative purposes here I adopt instead a much simpler and transparent procedure which yet gives convincing evidence that the OPE is in a good shape, and elucidates the actual root of alleged problems. Let me assume a rather arbitrary choice $m_b = 4.595 \text{ GeV}$, $m_c = 1.15 \text{ GeV}$, $\mu_\pi^2 = 0.45 \text{ GeV}^2$, $\tilde{\rho}_D^3 = 0.06 \text{ GeV}^3$ and $\rho_{LS}^3 = -0.15 \text{ GeV}^3$, natural from the theoretical viewpoint. The precise values of the quark masses are adjusted, however to literally accommodate $\langle M_X^2 \rangle_{E_\ell > 1 \text{ GeV}}$ now well measured by BaBar and CLEO. We then obtain

$$\begin{aligned} \langle M_X^2 \rangle &\simeq 4.434 \text{ GeV}^2 && [\text{cf. } (4.542 \pm 0.105) \text{ GeV (DELPHI)}] \\ \langle M_X^2 \rangle_{E_\ell > 1.5 \text{ GeV}} &\simeq 4.177 \text{ GeV}^2 && [\text{cf. } 4.180 \text{ GeV}^2 \text{ (BaBar), } 4.189 \text{ GeV}^2 \text{ (CLEO)}] \\ \langle E_\ell \rangle &\simeq 1.389 \text{ GeV} && [\text{cf. } (1.383 \pm 0.015) \text{ GeV (DELPHI)}] \\ \langle E_\gamma \rangle_{E_\gamma > 2 \text{ GeV}} &\simeq 2.329 \text{ GeV} && [\text{cf. } (2.346 \pm 0.034) \text{ GeV (CLEO)}] \\ \langle E_\gamma^2 - \bar{E}_\gamma^2 \rangle_{E_\gamma > 2 \text{ GeV}} &\simeq \frac{0.0202}{0.0233} \text{ GeV}^2 && [\text{cf. } (0.0226 \pm 0.0066 \pm 0.0020) \text{ GeV}^2 \text{ (CLEO)}] \end{aligned} \quad (12)$$

The moments of the photon spectra have been calculated including the perturbative corrections as in Ref. [21], i.e. they incorporate the above ‘exponential’ cut-related shifts at face value.¹ Their counterpart for the semileptonic decays has not been included here, however. I also show in Fig. 2 the E_{cut}^ℓ -dependence of $\langle M_X^2 \rangle$ as it comes from the literal OPE expressions with the above values of parameters. Experimental data are from Refs. [18, 22, 23, 24].

A quick glance shows that there is hardly any disagreement with theory, for both semileptonic and $b \rightarrow s + \gamma$ channels. Moreover, even the E_{cut}^ℓ -dependence is naturally reproduced. This radically differs from the results reported in [25]. I believe that what we have heard from Ligeti in this respect at this conference about the alleged problems for the OPE in describing the data, is not true.

¹The two values for the second E_γ -moment correspond to different light-cone ansätze; they are obtained discarding higher-order power corrections to the light-cone distribution function.

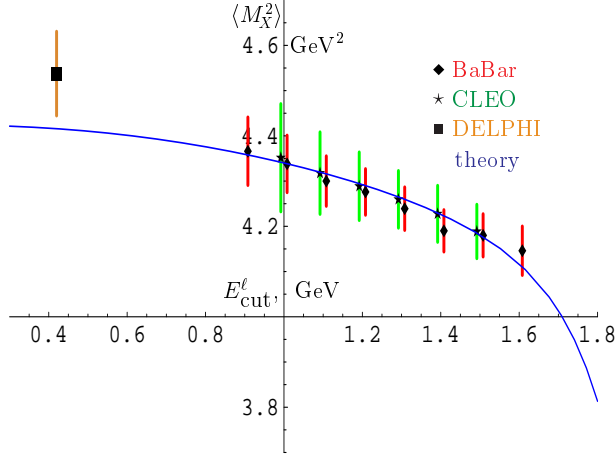


Figure 2: Experimental values of the average hadronic mass square $\langle M_X^2 \rangle$ in $B \rightarrow X_c \ell \nu$ at different lower cuts in E_ℓ and the literal OPE prediction (blue curve) for the stated heavy quark parameters. The DELPHI point assumes no cut on E_ℓ .

As mentioned above, the first leptonic moment $\langle E_\ell \rangle$ with cut at 1.5 GeV, the CLEO's R_1 comes out slightly different: 1.776 GeV vs. 1.7810 GeV measured by CLEO. The difference is certainly significant if viewed experimentally. However, is it really, if examined by theory?

3.2.1 Cuts in semileptonic moments

It was argued in Ref. [17] that similar cut-related ‘exponential’ biases missed in the naive OPE applications affect the truncated moments in the semileptonic decays as well. Their description, even simplified is less transparent and would be more involved, though, being given by a more complicated and non-universal object rather than the light-cone heavy quark distribution. Although the numerical aspects are rather uncertain, it is understood that they can be significant: the effective hardness

$$\mathcal{Q}_{\text{sl}} \simeq m_b - E_{\text{cut}} - \sqrt{E_{\text{cut}}^2 + m_c^2} \quad (13)$$

at $E_{\text{cut}} = 1.5$ GeV is about 1.25 GeV [14], nearly the same as for $B \rightarrow X_s + \gamma$ with $E_\gamma \gtrsim 2$ GeV. Ref. [21] estimated that the exponential terms in semileptonic decays with $E_{\text{cut}} \simeq 1.5$ GeV can introduce effects of the same scale as shifting m_b upward by up to 25 to 30 MeV (assuming fixed m_c and other heavy quark parameters). This rule of thumb – although very tentative – is useful to get an idea of the ultimate theoretical accuracy one can count on here.

For example, the CLEO's cut moment $R_1 = \langle E_\ell \rangle_{E_\ell > 1.5 \text{ GeV}}$ is approximately given by [14]

$$R_1 = 1.776 \text{ GeV} + 0.27(m_b - 4.595 \text{ GeV}) - 0.17(m_c - 1.15 \text{ GeV}) \quad \text{at } |V_{ub}/V_{cb}| = 0.08 \quad (14)$$

(the above mentioned values of the nonperturbative parameters are assumed). An increase in m_b by only 20 MeV would then change

$$R_1 \rightarrow R_1 + 0.0055 \text{ GeV} \quad (15)$$

perfectly fitting the central CLEO's value 1.7810 GeV. It is worth noting that the above equations make it evident that the imposed cut on E_ℓ degrades theoretical calculability of

R_1 far beyond its experimental error bars, the fact repeatedly emphasized over the last year. Unfortunately, this was not reflected in the fits of parameters which placed much weight on the values of $R_0 - R_2$ just owing to their small experimental uncertainties, whilst paying less attention to actual theoretical errors.

Even stronger reservations should be applied to the theoretical predictability for CLEO's R_2 representing the second moment with the cut, keeping in mind that the effective hardness deteriorates for higher moments.

The CLEO's ratio R_0 is the normalized decay rate with the cut on E_ℓ as high as 1.7 GeV, and for it hardness \mathcal{Q} is below 1 GeV. A precision – beyond just semiquantitative – treatment of nonperturbative effects is then questionable, and far more significant corrections should be allowed for.

We see that – contrary to what one could typically hear over the last year about experimental check of dynamical heavy quark theory – there is a good agreement of most data referring to sufficiently ‘hard’ decay distributions with the theory based on the OPE in QCD, if the ‘robust’ OPE approach is used. The latter was put forward [14] to get rid of unnecessary vulnerable assumptions of usually employed fits of the data. This consistency likewise refers to the absolute values of the heavy quark parameters necessary to accommodate the data. Theory itself reveals, however that the expansion becomes deceptive with an increase in the experimental cuts. Here I mentioned the clearest effects, those from the variety of ‘exponential’ terms in the effective hardness. While presently not amenable to precise theoretical treatment, they can be estimated using most natural assumptions and are found to be very significant for $E_{\text{cut}}^\ell \gtrsim 1.5 \text{ GeV}$ and $E_{\text{cut}}^\gamma \gtrsim 2 \text{ GeV}$ often employed in experiment. Taking these corrections at face value and incorporating in our predictions, we get a good, more than qualitative agreement with “less short-distance” inclusive decays as well.

A dedicated discussion of what may be wrong in a number of analyses claimed problems for the OPE goes far beyond the scope of the present talk. Two elements must be mentioned largely responsible for this, they have been emphasized early enough [20, 14], yet were mostly ignored. One is imposing unnecessary and dangerous constraint on the mass difference $m_b - m_c$ which relies on the too questionable $1/m_c$ expansion, and applying the corresponding power counting rules which are inadequate numerically. The second is blind usage of the naive OPE expressions where their validity is severely limited by barely sufficient effective hardness. The terms exponential in the latter – yet invisible in the naive approach – blow up there. Rephrasing Ben Bradley, one may then think that the alleged problems originate from “Dealing in the expressions, not necessarily in (OPE) truths”. I would also mention that using the proper Wilsonian version of the OPE with running quark masses and nonperturbative parameters is helpful in arriving at correct conclusions, and is probably indispensable for reaching the ultimate numerical precision in extracting the underlying parameters.

3.3 Semileptonic decays with $\tau\nu_\tau$

Experimental studies of the semileptonic decays $B \rightarrow X_c + \tau\nu_\tau$ are much more difficult, however even accurate measurements may be feasible at future high-luminosity super- B -factories. Relevant to physics discussed in Sect. 2, the $B \rightarrow D\tau\nu_\tau$ amplitude does not vanish at zero recoil

when $\vec{q} \rightarrow 0$ keeping the contribution of f_- (still suppressed by the τ mass). This opens an interesting way to check the BPS relation (2) and its accuracy dictated by the theorem (6). This would complement an independent determination of V_{cb} through the small-recoil $B \rightarrow D$ decay rate with light leptons.

Since the velocity range is rather limited for decays into τ leptons, $1 \leq w \leq 1.43$ the total $\text{BR}(B \rightarrow D \tau \nu_\tau)$ can be predicted in terms of $\varrho^2 - \frac{3}{4}$, at least in the BPS regime.

Inclusive decay width $\Gamma(B \rightarrow X_c + \tau \nu_\tau)$ is very sensitive to $m_b - m_c$, in contrast to $m_b - 0.6m_c$ for conventional decays (the same applies to decay distributions), which would finally provide a constraint on an alternative mass combinations.

On the other hand, energy release is limited here, $m_b - m_c - m_\tau \simeq 1.6 \text{ GeV}$. These decays therefore can be more sensitive to higher power corrections and represent a better candidate for studying local duality violations. One can also detect possible effects of the nonperturbative four-quark operators with charm quark, $\langle B | \bar{b}c \bar{c}b | B \rangle$; their potential role was emphasized in Ref. [5].

I am curious if it is possible to measure here separately the decay widths induced by vector and axial-vector currents. As emphasized in Sect. 2, this makes a big difference for the non-perturbative effects in the exclusive transitions to D and D^* , respectively. Yet for inclusive decays there should be no much difference once one enters the short-distance regime. Therefore, such analysis would provide interesting insights into the difference between the heavy quark expansion for inclusive and exclusive decays.

4 Conclusions

The theory of heavy quark decays is now a mature branch of QCD. Recent experimental studies of inclusive decays yielded valuable information crucial – through the comprehensive application of all the elements of the heavy quark expansion – for a number of exclusive decays as well. This signifies an important new stage in the heavy quark theory, since only a few years ago exclusive and inclusive decays were often viewed as largely separated, if not as antipodes theory-wise.

Generally speaking, there is ample evidence that heavy quark symmetry undergoes significant nonperturbative corrections for charm hadrons. However, there appears a class of practically relevant relations which remain robust. They are limited to the ground-state pseudoscalar B and D mesons, but do not include spin symmetry in the charm sector.

The accuracy of these new relations based on the proximity to the “BPS limit” strongly depends on the actual size of the kinetic expectation value in B mesons, $\mu_\pi^2(1 \text{ GeV})$. The experiment must verify it with maximal possible accuracy and reliability, without invoking ad hoc assumptions often made in the past with limited data available. This can be performed through the inclusive decays already in current experiments. If its value is confirmed not to exceed 0.45 GeV^2 , the $B \rightarrow D$ decays can be reliably treated by theory, and the estimate $\mathcal{F}_+(0) \simeq 1.03$ can provide a good alternative element in the comprehensive program of model-independent extraction of $|V_{cb}|$.

There are many other important consequences of the BPS regime. The slope of the IW function must be close to unity, and actually below it. The related constraints on the slope $\hat{\rho}^2$ of the experimentally observed combination of $B \rightarrow D^*$ formfactors should be incorporated in

the fits aiming at extrapolating the rate to zero recoil.

Contrary to what is often stated, the OPE-based theory of inclusive decays so far is in good shape when confronted experiment. A nontrivial consistency between quite different measurements, and between experiment and QCD-based theory, at the nonperturbative level, has been observed. Yet this refers to a thoughtful application of the OPE using the robust approach instead of doubtful approximations. There are good reasons to hope that new round of data will not reverse the tendency, and that the comprehensive approach shall indeed allow us to reach a percent level of reliable accuracy in translating $\Gamma_{sl}(B)$ to $|V_{cb}|$, as was proposed last year.

A final note – I think that the semileptonic decays with τ lepton and ν_τ have an interesting potential for both inclusive and exclusive decays at high-luminosity machines.

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